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 **Modified Newtonian Aerodynamics (rev.4)** 06 April 2016

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Introduction

 For vehicles travelling at supersonic speeds, the blunter parts of the vehicle can be locally hypersonic. The usual criterion for hypersonic flow, the local hypersonic similarity parameter, , is used. Drag on such body elements is usually estimated using Newtonian theory with the stagnation pressure coefficient proposed by Lees. Three body elements, a hemicylindrical fin leading edge, a hemispherical nose cap, and a cone are analyzed.

Notation

 Mnemonic Definition m

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 Free stream Mach number,

  Free stream Mach number normal to a fin leading

 edge,

  Dynamic pressure, lb/ft2

  Fineness ratio,

  Free stream ambient pressure, lb/ft2

  Stagnation pressure downstream of the normal shock,

   Pressure coefficient,

 **** Pressure coefficient on a conical body segment

  Cone base area ‒ forward face area, ft2

 Λ Leading edge sweep back angle, rad

 Angle between the normal to the local surface and the

 remote velocity vector, rad

  Maximum value of  at the base of the nose cap, rad

****Angle of attack, rad,

  Cone half angle, rad,

 ****Circumferential coordinate, rad

 ****  Aerodynamic reference area, ft2

  Aerodynamic reference length, ft,

  Drag force, lb,

N Normal Force, lb

**\_\_\_\_** Mnemonic Definition\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

  Drag coefficient,

 ****Normal force coefficient slope, per rad,

 **** Nose center of pressure, distance behind the nose tip,

 ft,

  Radius of the nose cap or leading edge, ft, and

 ****  Length of the fin leading edge, ft

  Length of a cone, ft.

Stagnation Pressure Coefficient

 First, consider the hemispherical nose cap. The streamline impacting the nose stagnation point passes through a normal shock before being isentropically compressed to zero speed. Equation (100) of ref. (1) is the Rayleigh pitot formula. It states that

  (100)

The stagnation pressure coefficient is, by definition,

  (1)

This equation is plotted below:



So long as , the stagnation pressure coefficient for a hemispherical cap merely requires that we substitute the Rayleigh pitot equation into the pressure coefficient definition.

 For a hemicylindrical leading edge, things are only a little more complicated. First, the Mach number normal to a sweep leading edge is

 Λ. (2)

Again, as long as **,** the cross flow stagnation pressure coefficient may be found from

  (3)

Classical Newtonian Pressure Coefficient

 Classical Newtonian theory is sometimes called impact theory because it is derived by assuming a column of air moving normal to the surface is brought to a complete stop relative to the surface. Impulse-momentum considerations then readily lead to

  (4)

The modified theory consists of replacing the constant  appearing above with the stagnation pressure coefficient .

**θ**

 **θM**

**R**

Figure 1 Hemispherical Nose Cap

Hemispherical Nose Cap

 `Begin by defining the nose cap drag coefficient as

 . (5)

An alternative definition of the drag force is the surface integral of the axial flow component of  Note that the surface integral of vanishes by Archimedes’ law. For the nose cap,

  or (6)

When multiplied by one half the stagnation pressure coefficient, we have the modified result we seek for the drag area of a hemispherical nose cap.

  (7)

 To find the hemispherical nose cap normal force slope, first consider a ring of arc length  and circumferential extent ****The Newtonian pressure coefficient acting on the area element at modest angle of attack is

  (8)

or, when  is infinitesimally small,

  (9)

Next, the normal force acting on the ring of length  is

 . (10)

Carrying out the circumferential  integration results in

  (11)

A second integration from  to  results in

  (12)

Expressed as a normal force coefficient corrected for the correct stagnation pressure, this becomes

 . (13)

The corresponding pitching moment about the forward tip point of the hemisphere is

  , or (14)

 **** (15)

The moment coefficient slope is

  (16)

and the center of pressure, measured from the forward tip point, is

 . (17)

Hemiscylindrical Leading Edge

 The hemicylindrical leading edge  estimate follows similar lines. Consider a piece of leading edge one unit in length. Then, assuming a flat plate airfoil,

 ,or (18)

 . (19)

This is the drag area normal to the leading edge, referenced to the cross flow dynamic pressure. Referenced to free stream, it is

 . (20)

This is the force normal to the leading edge. The component in the free stream direction is just

 . (21)

Thus, the leading edge drag area referenced to  , is just one half the cross flow stagnation pressure coefficient times the above result.

 , (22)

where  is based on the cross flow Mach number .

These results have been implemented in an EXCEL spread sheet called NEWTON.xls. A sample nose cap drag area plot for a nose cap radius of 0.1ft and maximum slope of 80 deg is shown below:



 Note that the Newtonian model is not valid for subsonic Mach numbers. But, according to ref. (2), page 15-6, the critical free stream Mach number for a sphere is about 0.6. This suggests that a practical engineer would fair the low speed end of this curve back to zero at a Mach number of 0.6.



The last graph is an example of the leading edge wave drag acting on a single flat plate fin panel 1/8” thick, 15” long, and swept at 30o. No data is reported for free stream Mach numbers less than 1.155 because the Newtonian theory does not apply to subsonic leading edges. As for the spherical nose cap, the critical cross flow Mach number for a circular cylinder is about 0.47. The subsonic leading edge drag can be faired in by eye.

Cone

 It's easy to show that the zero cone pressure coefficient is

 . (23)

The cone drag is then

 . (24)

If the cone has a small angle of attack, its local pressure coefficient can be found from eq.(9) while noting that :

 , or (25)

focusing on the normal force,

 . (26)

Substituting from eq.(25) into eq.(26) and integrating over  gives

 . (27)

If we have a cone of length the normal force coefficient slope is

 . (28)

The cone center of pressure is found a distanceaft of the nose tip.

Flat Face Aerodynamics

 Flat faced shapes are very commonly encountered, especially in conjunction with large angles of attack. First, Ref. 5 provides a comparison with experiment over a wide range of supersonic Mach numbers. The wind tunnel data therein all lie within 4% of Equation (1). The methods of Ref. (6), Chapter 15, suggest a subsonic model of the form

$ C\_{D}=0.85(1+ ¼\left(M^{2}+M^{4}\right))$ (29)

The result of Eq’s. (1) and (29) plotted together are shown below:

 Next, The data clearly show that

$ C\_{Nα}=0$ (30)

Even so, there an initial static pitching moment due to axial force when the stagnation point shifts with angle of attack. It turns out that the data are only good enough to enable derivation of an average stagnation point offset. It is

 $\frac{ð\frac{Offset}{BAse Dia}}{ðα}=-0.0054 per radian$ (31)

The negative sign indicates that a simple broadside disk is statically stable.

Interface to Second Order Shock Expansion Theory

 Reference 3. develops a marching procedure by which the surface pressure and lift distributions along a pointed body of revolution can be estimated.. However, this procedure requires the first body element to be a pointed cone with an attached shock, a condition not always satisfied in practice. The solution suggested here is to use the modified Newtonian described above to provide the nose tip drag, normal force and center of pressure. Assuming the hemisphere is tangent to the first conical element, its second order shock expansion solution can be estimated using the results of ref. (3) and the Newtonian results at the base of the nose tip:

**θM = **,

**p1 = p∞ + (½ p∞****M∞2) CpT cos2θM**,

**(∂p/∂s)1 = – 2 (½ p∞M∞2) CpT sinθM cosθM / R**.

The local Mach number can be estimated from the local pressure since the near-surface flow will expand isentropically from the stagnation point:

**p1 / pT = [ 1 + ((γ – 1) / 2) M12]–γ/(γ – 1)**

A brief inspection shows that this model clearly cannot tell the whole story. The **1/R** term in the equation for **(∂p/∂s)1** becomes very large fora tiny nose cap. Reference 4 provides the resolution First, define a dimensionless parameter **P**:

**P ≡ 2R/d**

The model just derived should apply when **P→1**, that is, when the hemispherical cap diameter approaches the body reference length (diameter). But when **P→0**, or when the cap radius is very small, the appropriate limiting conditions are:

**p1 = p∞ + (½ p∞M∞2)CPC**

**(∂p/∂s)1 = 0,** and

**M1 = M∞**

This model is equivalent to calculation the second order shock expansion flow neglecting the hemispherical except as an ex post-calculation for the wave drag and normal force that are added in with all the other body element contributions. Using the second Cierva interpolator in ref. (4) results in

 Finally, note that the limit of the surface pressure gradient estimated from Newtonian theory above will have an infinitely high value as the nose cap becomes very small. But, actually, as the hemisphere radius becomes very small, the surface pressure gradient must vanish as is appropriate for any pointed nose tip. Since most hemispherical nose caps are small, take **(∂p/∂s)1 = 0**. In the same vein, the **M1** value estimated above should not be used for small hemispherical caps. Instead, just use the free stream Mach number **M∞** for the initial condition on the foremost conical segment.

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